A

Synopsis

On

A Study of Random Solution and it's Characterizations for Random Differential and Integral Equations

Submitted to

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For the degree of

Doctor of Philosophy in Mathematics

Under

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Ph.D. Synopsis/Work Plan Entitled

A Study of Random Solution and its Characterizations for Random Differential and Integral Equations

Origin of the Research Problem :

The nondeterministic nature of phenomena in the general areas of biological, engineering, oceanographic and Physical Sciences the mathematical descriptions of such phenomena frequently result in random equations. It is to present theoretical results concerning certain classes of random equations and then to apply those results to problems that arise in the general areas just mentioned. In order to understand better the importance of developing such a theory and its applications, it is of interest to consider first the various ways in which these equations may arise.

Usually, the mathematical models or the equations that have been used to describe a particular phenomenon or process of the universe contains a parameter, which has some specific physically interpretations but whose values is not known. If any such phenomena involving mentioned parameters and which satisfy certain probabilistic and measure theoretic behavior with respect to this parameter, then we say it is a random phenomena. For example, in the theory of diffusion or heat conduction, we have the diffusion coefficient or the coefficient of conductivity that play the p rominent role in this phenomenon. Similarly in the case of wave theory, the propagation coefficient and in the theory of elasticity, the modulus of elasticity play the significant role in the behavior of the underlined processes. Thus, the coefficients or parameters that have an important role to play in the natural and physical phenomena like above are called random parameters.

The mathematical equations are solved using as the value of the parameter or coefficients, the mean value of the set of observations experimentally obtained. However, if the experiment is performed repeatedly, then the mean values found will vary ,and if the variation is large. The mean value actually used may be quite unsatisfactory. Thus, in practice the physical constant is not really a constant, but a random variable whose behavior is governed by some probability distribution. It is thus advantegeous to view these equations as being random rather than deterministic ,to search random solution and to study its statistical properties. we take about some parameters or coefficients, the random analysis of the random equations is inevitable and so the random equations have been studied in the literature since long time, by various mathematicians all over the world.

Review of Literature :

There are many ways in which random differential equations arise. Random differential equations appear uite naturally in the study of diffusion processes and Brownian motion[Gikhmann and Skorokhod].The classical Ito random integral equation may be found in many texts, for example, Doob. Integral equations with random kernels arise in random eigen value problems[Bharucha -Reid]. Random integral equations describe wave propogation in random media and the total number of conversation held at a given time in telephone traffic theory and Padgett and Tsokos.

The following five basic classes of random equations in the subject of random analysis.

I. Random algebraic equations ,
II. Random difference equations,
III. Random differential equations,
IV. Random integral equations,
V. Random integro-differential equations.

The random differential equations appears quite naturally in the study of changes or the rate of change of any processes like diffusion process or Brownian motion and such other like phenomenon, so the random differential equations have been the subject of rather extensive research among the several eminent mathematicians like soong in Random Differential Equations in Sciences and Engineering, Academic Press, New York (1973), Ladde and Lakhsmikantam, Random differential Inequalities, Academic press, New York (1980) Bharucha Reid in Fixed point the theorems in Probabilistic analysis, Bull. Amer. Math. Soc. 82 (1986), 611-645 and random Integral Equations, Academic Press, New York (1972) and Tsokos and Padgett in Random Integral Equations with application to life Sciences and Engineering, Academic press (1974) etc..Many researchers work in this direction and written a research papers on this topic by H. Engl,Omid S. Fard, V. Lakshmikantham, B.C. Dhage, D. S. Palimkar reference there in. The random equations have been utilized as models for a wide variety of random problems that have been encountered time and often in the areas of applied mathematical Sciences like Sciences and Environmental Sciences. The topic of differential equations is quite old as that of Famous Mathematician Isak Newton. It is much developed subject in both pure as well as applied branches of mathematics. In the beginning, the subject of differential equations was considered to be a part of applied mathematics, however in the latest stage, in has become most pure part of the pure mathematics.

Objective

The study of random differential equations is more than the study of classical differential equations and it is applicable in classical, random natural or physical phenomena of universe. The present work highlights of the following.

1. To study random differential and integral equations under existence, uniqueness and positivity of random solution.
2. To study random impulsive differential and integral equations under existence ,positivity of random solution.
3. To study various behaviors of random solution such as asymptoticity, stability and attractivity results.
4. Applications in real life problem .

Methodology

Random fixed point theory are the most powerful one which on application directly yields the existence of random solution and other characterization of random solution through .

(i) Geometrical fixed point theory: The geometric fixed point theory has been very famous and often times used in the study of nonlinear random differential and integral equation. The polish mathematician Stefan Banach proved contraction mapping principle in the year 1822. This is useful for proving the existence as well as uniqueness results for the nonlinear problems.

(ii) Algebraic fixed point theory: The main topological fixed point theorems has been used in the applications of nonlinear problems (1) Schauder fixed point principle (2) Kransnoselski fixed point principle and some others.

(iii) Topological fixed point theory: A fixed point theorem involving the mixed hypotheses from algebra, geometry and topology is called a hybrid fixed point theorem in abstract spaces. The study of hybrid fixed point theory is initiated by a Russian mathematician Krasnoselskii in the year 1964 and proved a fixed point theorem which combines the Banach fixed point theorem together with the Schauder fixed point theorem on subsets of a Banach space. This is useful for proving existence ,Stability and attractivity results etc..

Importance of Study

The present work is new to the literature and results of this work will be a good contribution to the study of random differential and integral equations.

The work is a completely original and will be a very good contribution in the area of nonlinear random differential and integral equations.

Since the random initial or boundary value problems are the generalizations of classical initial or boundary value problems, our results include general known results on deterministic initial or boundary value problems as special cases, when the random parameter is deleted . Again, since the natural and physical phenomena, heavily rests on the random parameters, the behavior of the processes can be functioned as per our win, by controlling the random parameters in a suitable way. It is claimed that the results of this work will from a basic, new and original contribution to the subject of nonlinear of random initial or boundary value problems , this work will be useful to Scientist, Engineers and Mathematicians etc., who are analyzing universal random phenomena covered by the nonlinear random initial or boundary value problems of random differential and integral equations .

Throughout this work, we have made use of some well- known and some new topological as well as some algebraic random fixed point theorems. Thus, our approach is helpful to unifying the theory of random differential equations via the concepts of nonlinear probabilistic functional analysis. Generally, a random initial or boundary value problems is together with initial or some boundary conditions, using the Green's function and superposition principle, the random initial or boundary value problems can be put in the form of a random operator equation .Then, we can apply a topological or a algebraic random fixed point theorems under suitable conditions to yield the desired result. Thus, our method to approach the random initial or boundary value problems are theoretical as well as constructive .

Year – wise Plan of Work

First Year

Survey and collection of basic materials required for this topic and field work. This includes collection of journals, Books and some research papers related to the topic. To study collected materials and unifying the direction of the topic.

Second Year

The collected material will be processed, analyzed on direction of the research topic. To study processed materials through direction of the research topic and attempt will be created. The created outcome will be presented in national and international gathering such as seminars, workshops. Conferences etc.

Third Year

The outcome result will be presented in national and international gathering such as seminars, workshops , conferences etc. Also will be published the outcome results in national, international journals during this year. Drafting, Writing and submission of final research work done report to Swami Ramanand Teerth Marathwada University ,Nanded. **References**

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Place : Nanded
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Research Students **Research Guide**
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